2)

$$y'' = -48(9-4x)^2(-4) = 192(9-4x)^2$$
$$y''(2\tfrac{1}{4}) = 192(0)^2 = 0 \qquad \text{test inconclusive}$$
$$y''' = 384(9-4x)(-4) = -1536(9-4x)$$
$$y'''(2\tfrac{1}{4}) = -1536(0) = 0 \qquad \text{test inconclusive}$$
$$y^{(4)} = 6144$$
$$y^{(4)}(2\tfrac{1}{4}) = 6144 > 0 \qquad \text{convex, relative minimum}$$

MARGINAL, AVERAGE, AND TOTAL CONCEPTS

4.10. Find (1) the marginal and (2) the average functions for each of the following total functions. Evaluate them at $Q = 3$ and $Q = 5$.

a) $\text{TC} = 3Q^2 + 7Q + 12$

1) $\text{MC} = \dfrac{d\text{TC}}{dQ} = 6Q + 7$

At $Q = 3$, $\text{MC} = 6(3) + 7 = 25$
At $Q = 5$, $\text{MC} = 6(5) + 7 = 37$

2) $\text{AC} = \dfrac{\text{TC}}{Q} = 3Q + 7 + \dfrac{12}{Q}$

At $Q = 3$, $\text{AC} = 3(3) + 7 + \frac{12}{3} = 20$
At $Q = 5$, $\text{AC} = 3(5) + 7 + \frac{12}{5} = 24.4$

Note: When finding the average function, be sure to divide the constant term by Q.

b) $\pi = Q^2 - 13Q + 78$

1) $\dfrac{d\pi}{dQ} = 2Q - 13$

At $Q = 3$, $\dfrac{d\pi}{dQ} = 2(3) - 13 = -7$

At $Q = 5$, $\dfrac{d\pi}{dQ} = 2(5) - 13 = -3$

2) $\text{A}\pi = \dfrac{\pi}{Q} = Q - 13 + \dfrac{78}{Q}$

At $Q = 3$, $\text{A}\pi = 3 - 13 + \frac{78}{3} = 16$
At $Q = 5$, $\text{A}\pi = 5 - 13 + \frac{78}{5} = 7.6$

c) $\text{TR} = 12Q - Q^2$

1) $\text{MR} = \dfrac{d\text{TR}}{dQ} = 12 - 2Q$

At $Q = 3$, $\text{MR} = 12 - 2(3) = 6$
At $Q = 5$, $\text{MR} = 12 - 2(5) = 2$

2) $\text{AR} = \dfrac{\text{TR}}{Q} = 12 - Q$

At $Q = 3$, $\text{AR} = 12 - 3 = 9$
At $Q = 5$, $\text{AR} = 12 - 5 = 7$

d) $\text{TC} = 35 + 5Q - 2Q^2 + 2Q^3$

1) $\text{MC} = \dfrac{d\text{TC}}{dQ} = 5 - 4Q + 6Q^2$

At $Q = 3$, $\text{MC} = 5 - 4(3) + 6(3)^2 = 47$
At $Q = 5$, $\text{MC} = 5 - 4(5) + 6(5)^2 = 135$

2) $\text{AC} = \dfrac{\text{TC}}{Q} = \dfrac{35}{Q} + 5 - 2Q + 2\text{Q}^2$

At $Q = 3$, $\text{AC} = \frac{35}{3} + 5 - 2(3) + 2(3)^2 = 28.67$
At $Q = 5$, $\text{AC} = \frac{35}{5} + 5 - 2(5) + 2(5)^2 = 52$

4.11. Find the marginal expenditure (ME) functions associated with each of the following supply functions. Evaluate them at $Q = 4$ and $Q = 10$.

a) $P = Q^2 + 2Q + 1$

To find the ME function, given a simple supply function, find the total expenditure (TE) function and take its derivative with respect to Q.

$$\text{TE} = PQ = (Q^2 + 2Q + 1)Q = Q^3 + 2Q^2 + Q$$
$$\text{ME} = \frac{d\text{TE}}{dQ} = 3Q^2 + 4Q + 1$$

At $Q = 4$, $\text{ME} = 3(4)^2 + 4(4) + 1 = 65$. At $Q = 10$, $\text{ME} = 3(10)^2 + 4(10) + 1 = 341$.

b) $P = Q^2 + 0.5Q + 3$

$$\text{TE} = PQ = (Q^2 + 0.5Q + 3)Q = Q^3 + 0.5Q^2 + 3Q$$
$$\text{ME} = 3Q^2 + Q + 3$$

At $Q = 4$, $\text{ME} = 3(4)^2 + 4 + 3 = 55$. At $Q = 10$, $\text{ME} = 3(10)^2 + 10 + 3 = 313$.

4.12. Find the MR functions for each of the following demand functions and evaluate them at $Q = 4$ and $Q = 10$.

a) $Q = 36 - 2P$

$$P = 18 - 0.5Q$$
$$\text{TR} = (18 - 0.5Q)Q = 18Q - 0.5Q^2$$
$$\text{MR} = \frac{d\text{TR}}{dQ} = 18 - Q$$

At $Q = 4$, $\text{MR} = 18 - 4 = 14$

At $Q = 10$, $\text{MR} = 18 - 10 = 8$

b) $44 - 4P - Q = 0$

$$P = 11 - 0.25Q$$
$$\text{TR} = (11 - 0.25Q)Q = 11Q - 0.25Q^2$$
$$\text{MR} = \frac{d\text{TR}}{dQ} = 11 - 0.5Q$$

At $Q = 4$, $\text{MR} = 11 - 0.5(4) = 9$

At $Q = 10$, $\text{MR} = 11 - 0.5(10) = 6$

4.13. For each of the following consumption functions, use the derivative to find the marginal propensity to consume $\text{MPC} = dC/dY$.

a) $C = C_0 + bY$

$$\text{MPC} = \frac{dC}{dY} = b$$

b) $C = 1500 + 0.75Y$

$$\text{MPC} = \frac{dC}{dY} = 0.75$$

4.14. Given $C = 1200 + 0.8Yd$, where $Yd = Y - T$ and $T = 100$, use the derivative to find the MPC.

When $C = f(Yd)$, make $C = f(Y)$ before taking the derivative. Thus,

$$C = 1200 + 0.8(Y - 100) = 1120 + 0.8Y$$
$$\text{MPC} = \frac{dC}{dY} = 0.8$$

Note that the introduction of a lump-sum tax into the income determination model *does not* affect the value of the MPC (or the multiplier).

4.15. Given $C = 2000 + 0.9Yd$, where $Yd = Y - T$ and $T = 300 + 0.2Y$, use the derivative to find the MPC.

$$C = 2000 + 0.9(Y - 300 - 0.2Y) = 2000 + 0.9Y - 270 - 0.18Y = 1730 + 0.72Y$$
$$\text{MPC} = \frac{dC}{dY} = 0.72$$

The introduction of a proportional tax into the income determination model *does* affect the value of the MPC and hence the multiplier.

4.16. Find the marginal cost functions for each of the following average cost functions.

a) $\text{AC} = 1.5Q + 4 + \dfrac{46}{Q}$

Given the average cost function, the marginal cost function is determined by first finding the total cost function and then taking its derivative, as follows:

$$\mathrm{TC} = (\mathrm{AC})Q = \left(1.5Q + 4 + \frac{46}{Q}\right)Q = 1.5Q^2 + 4Q + 46$$

$$\mathrm{MC} = \frac{d\mathrm{TC}}{dQ} = 3Q + 4$$

b) $\mathrm{AC} = \dfrac{160}{Q} + 5 - 3Q + 2Q^2$

$$\mathrm{TC} = \left(\frac{160}{Q} + 5 - 3Q + 2Q^2\right)Q = 160 + 5Q - 3Q^2 + 2Q^3$$

$$\mathrm{MC} = \frac{d\mathrm{TC}}{dQ} = 5 - 6Q + 6Q^2$$

OPTIMIZING ECONOMIC FUNCTIONS

4.17. Maximize the following total revenue TR and total profit π functions by (1) finding the critical value(s), (2) testing the second-order conditions, and (3) calculating the maximum TR or π.

a) $\mathrm{TR} = 32Q - Q^2$

1) $\mathrm{TR}' = 32 - 2Q = 0$

$Q = 16$ critical value

2) $\mathrm{TR}'' = -2 < 0$ concave, relative maximum

3) $\mathrm{TR} = 32(16) - (16)^2 = 256$

Note that whenever the value of the second derivative is negative over the whole domain of the function, as in (2) above, we can also conclude that the function is strictly concave and at a global maximum.

b) $\pi = -Q^2 + 11Q - 24$

1) $\pi' = -2Q + 11 = 0$

$Q = 5.5$ critical value

2) $\pi'' = -2 < 0$ concave, relative maximum

3) $\pi = -(5.5)^2 + 11(5.5) - 24 = 6.25$

c) $\pi = -\frac{1}{3}Q^3 - 5Q^2 + 2000Q - 326$

1) $\pi' = -Q^2 - 10Q + 2000 = 0$ (*4.1*)

$-1(Q^2 + 10Q - 2000) = 0$ (*4.2*)

$(Q + 50)(Q - 40) = 0$

$Q = -50 \qquad Q = 40$ critical values

2) $\pi'' = -2Q - 10$

$\pi''(40) = -2(40) - 10 = -90 < 0$ concave, relative maximum

$\pi''(-50) = -2(-50) - 10 = 90 > 0$ convex, relative minimum

Negative critical values will subsequently be ignored as having no economic significance.

3) $\pi = -\frac{1}{3}(40)^3 - 5(40)^2 + 2000(40) - 326 = 50{,}340.67$

Note: In testing the second-order conditions, as in step 2, always take the second derivative from the original first derivative (*4.1*) before any *negative* number has been factored out. Taking the second derivative from the first derivative after a negative has been factored out, as in (*4.2*), will reverse the second-order conditions and suggest that the function is maximized at $Q = -50$ and minimized at $Q = 40$. Test it yourself.

d) $\pi = -Q^3 - 6Q^2 + 1440Q - 545$

1) $\pi' = -3Q^2 - 12Q + 1440 = 0$
$-3(Q - 20)(Q + 24) = 0$
$Q = 20 \qquad Q = -24 \qquad$ critical values

2) $\pi'' = -6Q - 12$
$\pi''(20) = -6(20) - 12 = -132 < 0 \qquad$ concave, relative maximum

3) $\pi = -(20)^3 - 6(20)^2 + 1440(20) - 545 = 17{,}855$

4.18. From each of the following total cost TC functions, find (1) the average cost AC function, (2) the critical value at which AC is minimized, and (3) the minimum average cost.

a) $\text{TC} = Q^3 - 5Q^2 + 60Q$

1) $\text{AC} = \dfrac{\text{TC}}{Q} = \dfrac{Q^3 - 5Q^2 + 60Q}{Q} = Q^2 - 5Q + 60$

2) $\text{AC}' = 2Q - 5 = 0 \qquad Q = 2.5$
$\text{AC}'' = 2 > 0 \qquad$ convex, relative minimum

3) $\text{AC}(2.5) = (2.5)^2 - 5(2.5) + 60 = 53.75$

Note that whenever the value of the second derivative is positive over the whole domain of the function, as in (2) above, we can also conclude that the function is strictly convex and at a global minimum.

b) $\text{TC} = Q^3 - 21Q^2 + 500Q$

1) $\text{AC} = \dfrac{Q^3 - 21Q^2 + 500Q}{Q} = Q^2 - 21Q + 500$

2) $\text{AC}' = 2Q - 21 = 0 \qquad Q = 10.5$
$\text{AC}'' = 2 > 0 \qquad$ convex, relative minimum

3) $\text{AC} = (10.5)^2 - 21(10.5) + 500 = 389.75$

4.19. Given the following total revenue and total cost functions for different firms, maximize profit π for the firms as follows: (1) Set up the profit function $\pi = \text{TR} - \text{TC}$, (2) find the critical value(s) where π is at a relative extremum and test the second-order condition, and (3) calculate the maximum profit.

a) $\text{TR} = 1400Q - 6Q^2 \qquad \text{TC} = 1500 + 80Q$

1) $\pi = 1400Q - 6Q^2 - (1500 + 80Q)$
$= -6Q^2 + 1320Q - 1500$

2) $\pi' = -12Q + 1320 = 0$
$Q = 110 \qquad$ critical value

$\pi'' = -12 < 0 \qquad$ concave, relative maximum

3) $\pi = -6(110)^2 + 1320(110) - 1500 = 71{,}000$

b) $\text{TR} = 1400Q - 7.5Q^2 \qquad \text{TC} = Q^3 - 6Q^2 + 140Q + 750$

1) $$\begin{aligned} \pi &= 1400Q - 7.5Q^2 - (Q^3 - 6Q^2 + 140Q + 750) \\ &= -Q^3 - 1.5Q^2 + 1260Q - 750 \end{aligned} \tag{4.3}$$

2) $$\begin{aligned} \pi' &= -3Q^2 - 3Q + 1260 = 0 \\ &= -3(Q^2 + Q - 420) = 0 \\ &= -3(Q+21)(Q-20) = 0 \end{aligned}$$

$$Q = -21 \qquad Q = 20 \qquad \text{critical values}$$

Take the second derivative directly from (*4.3*), as explained in Problem 4.17(*c*), and ignore all negative critical values.

$$\pi'' = -6Q - 3$$
$$\pi''(20) = -6(20) - 3 = -123 < 0 \qquad \text{concave, relative maximum}$$

3) $\pi = -(20)^3 - 1.5(20)^2 + 1260(20) - 750 = 15{,}850$

c) $\text{TR} = 4350Q - 13Q^2 \qquad \text{TC} = Q^3 - 5.5Q^2 + 150Q + 675$

1) $$\begin{aligned} \pi &= 4350Q - 13Q^2 - (Q^3 - 5.5Q^2 + 150Q + 675) \\ &= -Q^3 - 7.5Q^2 + 4200Q - 675 \end{aligned}$$

2) $$\begin{aligned} \pi' &= -3Q^2 - 15Q + 4200 = 0 \\ &= -3(Q^2 + 5Q - 1400) = 0 \\ &= -3(Q+40)(Q-35) = 0 \end{aligned}$$

$$Q = -40 \qquad Q = 35 \qquad \text{critical values}$$

$$\pi'' = -6Q - 15$$
$$\pi''(35) = -6(35) - 15 = -225 < 0 \qquad \text{concave, relative maximum}$$

3) $\pi = -(35)^3 - 7.5(35)^2 + 4200(35) - 675 = 94{,}262.50$

d) $\text{TR} = 5900Q - 10Q^2 \qquad \text{TC} = 2Q^3 - 4Q^2 + 140Q + 845$

1) $$\begin{aligned} \pi &= 5900Q - 10Q^2 - (2Q^3 - 4Q^2 + 140Q + 845) \\ &= -2Q^3 - 6Q^2 + 5760Q - 845 \end{aligned}$$

2) $$\begin{aligned} \pi' &= -6Q^2 - 12Q + 5760 = 0 \\ &= -6(Q^2 + 2Q - 960) = 0 \\ &= -6(Q+32)(Q-30) = 0 \end{aligned}$$

$$Q = -32 \qquad Q = 30 \qquad \text{critical values}$$

$$\pi'' = -12Q - 12$$
$$\pi''(30) = -12(30) - 12 = -372 < 0 \qquad \text{concave, relative maximum}$$

3) $\pi = -2(30)^3 - 6(30)^2 + 5760(30) - 845 = 112{,}555$

4.20. Prove that marginal cost (MC) must equal marginal revenue (MR) at the profit-maximizing level of output.

$$\pi = \text{TR} - \text{TC}$$

To maximize π, $d\pi/dQ$ must equal zero.

$$\frac{d\pi}{dQ} = \frac{d\text{TR}}{dQ} - \frac{d\text{TC}}{dQ} = 0$$
$$\frac{d\text{TR}}{dQ} = \frac{d\text{TC}}{dQ}$$
$$\text{MR} = \text{MC} \qquad \text{Q.E.D.}$$

4.21. A producer has the possibility of discriminating between the domestic and foreign markets for a product where the demands, respectively, are

$$Q_1 = 21 - 0.1P_1 \tag{4.4}$$
$$Q_2 = 50 - 0.4P_2 \tag{4.5}$$

Total cost $= 2000 + 10Q$ where $Q = Q_1 + Q_2$. What price will the producer charge in order to maximize profits (*a*) with discrimination between markets and (*b*) without discrimination? (*c*) Compare the profit differential between discrimination and nondiscrimination.

a) To maximize profits under price discrimination, the producer will set prices so that MC = MR in each market. Thus, $\text{MC} = \text{MR}_1 = \text{MR}_2$. With $\text{TC} = 2000 + 10Q$,

$$\text{MC} = \frac{d\text{TC}}{dQ} = 10$$

Hence MC will be the same at all levels of output. In the domestic market,

$$Q_1 = 21 - 0.1P_1$$

Hence, $$P_1 = 210 - 10Q_1$$

$$\text{TR}_1 = (210 - 10Q_1)\,Q_1 = 210Q_1 - 10Q_1^2$$

and $$\text{MR}_1 = \frac{d\text{TR}_1}{dQ_1} = 210 - 20Q_1$$

When $\text{MR}_1 = \text{MC}$, $$210 - 20Q_1 = 10 \qquad Q_1 = 10$$

When $Q_1 = 10$, $$P_1 = 210 - 10(10) = 110$$

In the foreign market, $$Q_2 = 50 - 0.4P_2$$

Hence, $$P_2 = 125 - 2.5Q_2$$

$$\text{TR}_2 = (125 - 2.5Q_2)\,Q_2 = 125Q_2 - 2.5Q_2^2$$

Thus, $$\text{MR}_2 = \frac{d\text{TR}_2}{dQ_2} = 125 - 5Q_2$$

When $\text{MR}_2 = \text{MC}$, $$125 - 5Q_2 = 10 \qquad Q_2 = 23$$

When $Q_2 = 23$, $$P_2 = 125 - 2.5(23) = 67.5$$

The discriminating producer charges a lower price in the foreign market where the demand is relatively more elastic and a higher price ($P_1 = 110$) in the domestic market where the demand is relatively less elastic.

b) If the producer does not discriminate, $P_1 = P_2$ and the two demand functions (*4.4*) and (*4.5*) may simply be aggregated. Thus,

$$Q = Q_1 + Q_2 = 21 - 0.1P + 50 - 0.4P = 71 - 0.5P$$

Hence, $$P = 142 - 2Q$$

$$\text{TR} = (142 - 2Q)\,Q = 142Q - 2Q^2$$

and $$\text{MR} = \frac{d\text{TR}}{dQ} = 142 - 4Q$$

When MR = MC, $$142 - 4Q = 10 \qquad Q = 33$$

When $Q = 33$, $$P = 142 - 2(33) = 76$$

When no discrimination takes place, the price falls somewhere between the relatively high price of the domestic market and the relatively low price of the foreign market. Notice, however, that the quantity sold remains the same: at $P = 76$, $Q_1 = 13.4$, $Q_2 = 19.6$, and $Q = 33$.

c) With discrimination,

$$\text{TR} = \text{TR}_1 + \text{TR}_2 = P_1 Q_1 + P_2 Q_2 = 110(10) + 67.5(23) = 2652.50$$

$\text{TC} = 2000 + 10Q$, where $Q = Q_1 + Q_2$.

$$\text{TC} = 2000 + 10(10 + 23) = 2330$$

Thus, $$\pi = \text{TR} - \text{TC} = 2652.50 - 2330 = 322.50$$

Without discrimination,

$$\text{TR} = PQ = 76(33) = 2508$$

$\text{TC} = 2330$ since costs do not change with or without discrimination. Thus, $\pi = 2508 - 2330 = 178$. Profits are higher with discrimination (322.50) than without discrimination.

4.22. Faced with two distinct demand functions

$$Q_1 = 24 - 0.2P_1 \qquad Q_2 = 10 - 0.05P_2$$

where $\text{TC} = 35 + 40Q$, what price will the firm charge (*a*) with discrimination and (*b*) without discrimination?

a) With $Q_1 = 24 - 0.2P_1$,

$$\begin{aligned} P_1 &= 120 - 5Q_1 \\ \text{TR}_1 &= (120 - 5Q_1)Q_1 = 120Q_1 - 5Q_1^2 \\ \text{MR}_1 &= 120 - 10Q_1 \end{aligned}$$

The firm will maximize profits where $\text{MC} = \text{MR}_1 = \text{MR}_2$

$$\begin{aligned} \text{TC} &= 35 + 40Q \\ \text{MC} &= 40 \end{aligned}$$

When $\text{MC} = \text{MR}_1$, $$40 = 120 - 10Q_1 \qquad Q_1 = 8$$

When $Q_1 = 8$, $$P_1 = 120 - 5(8) = 80$$

In the second market, with $Q_2 = 10 - 0.05P_2$,

$$\begin{aligned} P_2 &= 200 - 20Q_2 \\ \text{TR}_2 &= (200 - 20Q_2)Q_2 = 200Q_2 - 20Q_2^2 \\ \text{MR}_2 &= 200 - 40Q_2 \end{aligned}$$

When $\text{MC} = \text{MR}_2$, $$40 = 200 - 40Q_2 \qquad Q_2 = 4$$

When $Q_2 = 4$, $$P_2 = 200 - 20(4) = 120$$

b) If the producer does not discriminate, $P_1 = P_2 = P$ and the two demand functions can be combined, as follows:

$$Q = Q_1 + Q_2 = 24 - 0.2P + 10 - 0.05P = 34 - 0.25P$$

Thus,

$$\begin{aligned} P &= 136 - 4Q \\ \text{TR} &= (136 - 4Q)Q = 136Q - 4Q^2 \\ \text{MR} &= 136 - 8Q \end{aligned}$$

At the profit-maximizing level, $\text{MC} = \text{MR}$.

$$40 = 136 - 8Q \qquad Q = 12$$

At $Q = 12$, $$P = 136 - 4(12) = 88$$

4.23. Use the MR = MC method to (*a*) maximize profit π and (*b*) check the second-order conditions, given

$$\text{TR} = 1400Q - 7.5Q^2 \qquad \text{TC} = Q^3 - 6Q^2 + 140Q + 750$$

a) $\text{MR} = \text{TR}' = 1400 - 15Q$, $\text{MC} = \text{TC}' = 3Q^2 - 12Q + 140$

Equate MR = MC. $\qquad 1400 - 15Q = 3Q^2 - 12Q + 140$

Solve for Q by moving everything to the right.

$$3Q^2 + 3Q - 1260 = 0$$
$$3(Q + 21)(Q - 20) = 0$$
$$Q = -21 \qquad Q = 20 \qquad \text{critical values}$$

b) $\text{TR}'' = -15 \qquad \text{TC}'' = 6Q - 12$

Since $\pi = \text{TR} - \text{TC}$ and the objective is to maximize π, be sure to *subtract* TC″ *from* TR″, or you will reverse the second-order conditions and select the wrong critical value.

$$\begin{aligned} \pi'' &= \text{TR}'' - \text{TC}'' \\ &= -15 - 6Q + 12 = -6Q - 3 \\ \pi''(20) &= -6(20) - 3 = -123 < 0 \qquad \text{concave, relative maximum} \end{aligned}$$

Compare these results with Problem 4.19(*b*).

THE MARGINAL RATE OF TECHNICAL SUBSTITUTION

4.24. An *isoquant* depicts the different combinations of inputs K and L that can be used to produce a specific level of output Q. One such isoquant for the output level $Q = 2144$ is

$$16K^{1/4}L^{3/4} = 2144$$

(*a*) Use implicit differentiation from Section 3.9 to find the slope of the isoquant dK/dL which in economics is called the *marginal rate of technical substitution* (MRTS). (*b*) Evaluate the marginal rate of technical substitution at $K = 256$, $L = 108$.

a) Take the derivative of each term with respect to L and treat K as a function of L.

$$\frac{d}{dL}(16K^{1/4}L^{3/4}) = \frac{d}{dL}(2144)$$

Use the product rule since K is being treated as a function of L.

$$16K^{1/4} \cdot \frac{d}{dL}(L^{3/4}) + L^{3/4} \cdot \frac{d}{dL}(16K^{1/4}) = \frac{d}{dL}(2144)$$

$$\left(16K^{1/4} \cdot \frac{3}{4}L^{-1/4}\right) + \left(L^{3/4} \cdot 16 \cdot \frac{1}{4}K^{-3/4} \cdot \frac{dK}{dL}\right) = 0$$

$$12K^{1/4}L^{-1/4} + 4K^{-3/4}L^{3/4} \cdot \frac{dK}{dL} = 0$$

Solve algebraically for dK/dL.

$$\frac{dK}{dL} = \frac{-12K^{1/4}L^{-1/4}}{4K^{-3/4}L^{3/4}} = \frac{-3K}{L}$$

b) At $K = 256$ and $L = 108$.

$$\text{MRTS} = \frac{dK}{dL} = \frac{-3(256)}{108} = -7.11$$

This means that if L is increased by 1 relatively small unit, K must decrease by 7.11 units in order to remain on the production isoquant where the production level is constant. See also Problem 6.51.